REAL OPTION VALUE

CHAPTER 8   SCALE OPTIONS

In the previous chapters, only a few subsequent management actions after the initial investments are considered in the real option format. Now we allow several types of management reaction to exogenous changes in value, price, operating cost, investment and scale costs, and to other factors.

8.1   SCALE MANAGEMENT ACTIONS

What is the optimal state (normal operations, contract, suspend and maintain, revert to normal or reduced service, abandon) given current (or estimated) profits for an asset owner, when future profits are variable? What is the optimal investment timing for a prospective asset owner, when there are several feasible state alternatives and irrecoverable costs of making these choices? The conventional methodology is to follow the microeconomic theory of optimal investment and identify the corresponding Marshallian (net present value) triggers for entry and exit based on net present values. An alternative is to solve the state and investment problems within a real options methodological framework.

Entry and exit (or abandon and scrap) opportunities are present in varying degrees in most industries. The classical examples are from shipping, where a prospective shipowner acquires an option to build (usually a ship type consistent with other owner characteristics or operations); then decides whether to exercise, sell or let expire that building option; once operational, the shipowner then has the option to reduce operational expense by slow sailing or mothballing if freight rates fall below operating costs, with the option to revert to “full service” operations, or scrap, if rates fall even more. Property downsizing problems and opportunities are created by a combination of low rents, low occupancy and high maintenance or operating costs for a property with fixed capacity. Examples are a hotel where the demand is seasonal; it may be feasible to operate at reduced service costs, before eventually closing the hotel. Cold weather climate seaside resorts are often completely closed and maintained during the winter season, rather than abandoned. Real estate in depressed markets or depressed areas or disaster places are sometimes boarded-up, and eventually if there is no recovery, abandoned or converted to other uses. Speculative office buildings without firm tenancies are sometimes constructed during up market cycles, but during down or very uncertain markets are left vacant.[1] Football stadium net income is dependent on the product of average ticket prices times tickets sold. When net income less costs becomes significantly negative, perhaps game or season cancellations are warranted. Airlines may shrink capacity, renegotiate employee costs, and reduce ticket prices when loads decline, seasonally, cyclically or suddenly. Natural resources (mines, petroleum) are brought into production, sometimes allowing for expansion, but more commonly at an appropriate initial maximum scale, which declines over time as mining seams become stretched, or petroleum pressure declines. Some mines and some petroleum production facilities can be contracted to save cost, but usually facilities are closed and maintained. The facilities might be re-opened if commodity prices increase and so exceed operating costs.

The classical models for operating and entry/exit options in entities with fixed capacity include Mossin (1968), Tourinho (1979), Brennan and Schwartz (1985) and Dixit (1989)[2]. These authors assume various stochastic processes for copper prices or shipping freight rates, as well as different possibilities and bounds of management actions. Assuming shipping revenues follow a stationary discrete random walk, Mossin (1968) derives optimal triggers for ship mothballing and reactivation. Tourinho (1979) provides a closed-form solution for perpetual entry and exit options, when prices have zero expected drift, but there are possible option holding costs (maintenance or lease costs on undeveloped petroleum

[1] Centrepoint in London is a famous example.
[2] Some of these models are summarized and applied in Dixit and Pindyck (1994).

reserves). Brennan and Schwartz (1985) assume that output prices follow a geometric Brownian motion process, but consider only investing in and operating a mine, or closing and maintaining, or abandonment. Dixit and Pindyck (1994), incorporating Dixit (1989), consider investing, mothballing, reactivation or abandonment; the order of options is from idle to active, with the exercise prices consisting of the initial construction costs, mothballing, reactivation and scrapping costs. Other authors have allowed for restricted reversibility, diminishing production capacity over time, redevelopments, multiple switches, and investment lags.

Assume that the annual revenue= price (“P”) per the initial investment cost (which for property might be annual rent times available space times occupancy) is viewed as stochastic, and other costs such as investment or abandonment are deterministic or constant. Then appropriate and sequentially ordered management actions are considered after the initial investment.

In section 8.3.1 a firm converts from being idle (with, however, a proprietary option to build), to investing in and operating a facility. If P subsequently falls, the next section 8.3.2 considers the option of abandonment, or switching to an alternative use. In section 8.4, if, after the initial investment, P falls, there is an option to contract, reducing costs (and reducing services and gross profits). If P then increases, the next option is to revert to full service. If P falls instead, the next option is abandonment, or switching to an alternative use. Thus up to four options are considered, each with trigger prices, exercise costs, and usually different gross profits and operating costs in each state. Other authors (see Paxson, 2005) consider even more management options (see Figure 8.9).

8.2. DETERMINISTIC ENTRY/EXIT INDICATORS

Consider a single discrete project with sunk investment cost K, and operating cost (entirely variable) C per unit of time. Define the output flow of the project as a unit, so the revenue for the project is simply the output price P. The optimal decision rule consists of two triggers P_K and P_A, with $P_K > P_A$ such that the investment should be made if P rises above P_K and abandoned if P falls to or below P_A.

Suppose that the firm does not have an investment in place and that it believes that P will never change after making the investment following Marshall (1890). It will invest if $W_K = P > (C + rK)$. The right-hand side is the annualised full cost of making (rK) and operating (C) the investment. Suppose now that a firm has such an investment in place and that the price falls to a new level P, where the firm believes it will persist forever. The firm will abandon if $W_A = P < C$. So the full cost serves as the entry trigger P_K and the variable cost as the exit trigger P_A.

Hysteresis (delay in reactions between investment and abandonment) can be explained by this theory. Suppose the price is initially between C and C + rK. If P increases to a level above C + rK, the firm will invest, following the Marshallian deterministic investment rule. Then after making the investment, even P falling to its original level would be insufficient to induce abandonment. Hysteresis occurs within the range of C and C +rK.

One example of problems of deterministic investment rules is the role of expectations of the price process. Suppose P is expected to be mean reverting over the long run to P* (between C and C + rK). Now a price of C + rK would not suffice to induce investment. P_K has to be higher than C + rK, so above normal prices for a while can compensate for below normal prices that might eventually follow.

8.3 STOCHASTIC MODEL

In order to formulate an appropriate framework for the model, a number of assumptions-restrictions need to be imposed. It is assumed that there is a single factor P (price per investment scale) which follows a geometric Brownian motion stochastic process:

$$\frac{dP}{P} = (\mu_P - \delta_P)dt + \sigma_P dz \tag{8.1}$$

where μ_P is the drift rate over time, δ_P is the asset yield or convenience yield, σ_P is the instantaneous standard deviation of the P disturbance, and dz is the standardized Wiener process. All of the costs involved are known and constant, and the riskless rate of interest (r) is fixed. Moreover, the options to alter states are perceived to be perpetual, since the asset is assumed to last forever. Finally, the investment cost represent costs of investing in a newly built asset, ignoring taxes and subsidies, and is considered irrecoverable, as are the one-off costs of existing (abandonment, scrapping or switching uses, usually positive costs in the former case, negative costs in the latter).

It is assumed that the stochastic evolution of P can be perfectly replicated from continuously trading in securities (with no transaction costs and perfect capital markets), and that portfolios of such securities are perfectly correlated with dz. (There are a wide variety of traded securitizations of gross profits from some types of assets, including hotels and properties.) The total expected rate of return on holding such replicating portfolios is μ = r- δ, where δ is the expected asset yield (or convenience yield on commodities). A contingent claim on P with the given profit flow will earn the same as the replicating portfolio, and thus the net position of a long position in the contingent claim and a short position of V'(P) (the first derivative of the function with respect to P) in the replicating portfolio will be riskless and so earn the riskfree rate of return.

The notation is as follows:

Input Variables

P = annual price per unit productive capacity, K;

C = annual operating cost;

K = investment cost;

A = abandonment cost (this cost may be negative, if the asset has an alternative use value in excess of demolition or conversion costs);

r, δ, and σ, are the riskless rate of interest, asset yield (net cost of carry[3]), and volatility, respectively, (assumed constant across states and time).

Output Variables:

$\beta_{1,2}$ = equations 8.6 and 8.7;

P_K = P threshold that justifies initial investment;

P_A = P threshold that justifies abandonment;

In many cases K, A >0 is required as a constraint, to rule out a "money making machine" of rapid cycles of investment and abandonment.

Making an investment is like exercising an option, with the cost of the investment equal to the strike price of the option. Typically, the asset that is acquired by exercising the option to invest includes another option, namely to abandon the investment and revert to the original situation. There are two interlinked option pricing problems.

In a stochastic model allowing for both investment and abandonment, there are two differential equations that the valuation functions must satisfy:

IDLE

$$\tfrac{1}{2}\sigma^2 P^2 V_0''(P) + (r-\delta)PV_0'(P) - rV_0(P) = 0 \qquad (8.2)$$

3. See Dixit and Pindyck (1994) (pp. 114-124) for a discussion of the conditions under which $\delta = r - \mu$, so that δ would be considered the rate of return shortfall on the gross profits drift, also deemed the net cost of carry. Note that the return on the firm V(P) with the embedded options to change states is assumed to be equivalent to the return on any replicating portfolio. Note some other authors require a specific risk aversion coefficient.

ACTIVE

$$\tfrac{1}{2}\sigma^2 P^2 V_K''(P) + (r-\delta)PV_K'(P) - rV_K(P) + P - C = 0 \quad (8.3)$$

The solutions for each of these equations are:

$$V_0(P) = A_1 P^{\beta_1} \quad (8.4)$$

$$V_K(P) = B_1 P^{\beta_1} + B_2 P^{\beta_2} + P/\delta - C/r \quad (8.5)$$

The general solution for each state is of the form of some constant (to be determined) times P to the power of $\beta_{1,2}$ given by:

$$\beta_1 = \tfrac{1}{2} - (r-\delta)/\sigma^2 + \sqrt{[(r-\delta)/\sigma^2 - \tfrac{1}{2}]^2 + 2r/\sigma^2} \quad >1 \quad (8.6)$$

$$\beta_2 = \tfrac{1}{2} - (r-\delta)/\sigma^2 - \sqrt{[(r-\delta)/\sigma^2 - \tfrac{1}{2}]^2 + 2r/\sigma^2} \quad <0 \quad (8.7)$$

Each of the actions must meet value matching and smooth pasting conditions.

8.3.1 ENTRY ONLY

In the one option framework, where there is no abandonment option, or where (C-rA)<0 that is the operating cost is always less than the annualized cost of abandonment, the option to invest will be exercised when the investment option is equal to the value of the optimal investment trigger less the operating cost less the investment cost, or $A_1 P_K^{\beta_1} = \frac{P_K}{\delta} - \frac{C}{r} - K$ (8.8)

Assuming that V(P) is continuous and smooth at the critical exercise trigger P_K , the first derivative of equation (8.8) with respect to P_K is $\beta_1 A_1 P_K^{\beta_1 - 1} = \frac{1}{\delta}$. (8.9)

Simplifying and and rearranging equations (8.8) and (8.9), the simple investment option solution is equation (8.4), with

$$P_K = \frac{\beta_1}{\beta_1 - 1}\left(\frac{C}{r} + K\right)\delta \qquad \text{and } A_1 = \frac{\frac{P_K}{\delta} - \frac{C}{r} - K}{P_K^{\beta_1}} \; . \tag{8.10}$$

$$ROV_0 = A_1 P^{\beta_1} = \left(\frac{P_K}{\delta} - \frac{C}{r} - K\right)\left(\frac{P}{P_K}\right)^{\beta_1} . \tag{8.11}$$

8.3.2 ENTRY and EXIT

Where there is the possibility of both entry (investment) and abandonment (exit at a cost A), represented by both equations (8.2) and (8.3) above, the solution for equation (8.3) is equation (8.5) except that B_1 equals zero, since it is assumed that once the price goes to zero, there is no longer an investment opportunity.

$$V_K(P) = B_2 P^{\beta_2} + P/\delta - C/r \tag{8.12}$$

The first term of equation (8.12) represents the value of the option to abandon, whereas the other two terms represent the perpetual net value of operating the asset. Now, there are four unknowns that need to be determined, namely the two optimal thresholds P_K and P_A, and the two option value coefficients A_1 and B_2. At the optimal entry point P_K, the value-matching and smooth pasting conditions need to be satisfied in addition to equations similar to equations (8.8) and (8.9). The optimal abandonment threshold P_A must satisfy:

$$V_K(P_A) = V_0(P_A) - A \qquad V_K'(P_A) = V_0'(P_A) \tag{8.13}$$

After substitutions and simplifications, there are four equations to be solved simultaneously.

$$A_1 P_K^{\beta_1} - B_2 P_K^{\beta_2} - P_K/\delta + C/r + K = 0 \tag{8.14}$$
$$\beta_1 A_1 P_K^{\beta_1 - 1} - \beta_2 B_2 P_K^{\beta_2 - 1} - 1/\delta = 0 \tag{8.15}$$

$$-A_1 P_A^{\beta_1} + B_2 P_A^{\beta_2} + P_A/\delta - C/r + A = 0 \tag{8.16}$$
$$-\beta_1 A_1 P_A^{\beta_1 - 1} + \beta_2 B_2 P_A^{\beta_2 - 1} + 1/\delta = 0 \tag{8.17}$$

When (C-rA)<0, the solutions (shown in the Appendix) for the four equations are:

$$P_K = \frac{\beta_1}{\beta_1 - 1}\frac{\delta}{r}(C + rK) \tag{8.18}$$

$$P_A = MAX[\frac{-\beta_2}{-\beta_2 - 1}\frac{\delta}{r}W_A, 0] \tag{8.19}$$

$$W_K = C + rK \tag{8.20}$$

$$W_A = MAX[(C - rA), 0] \tag{8.21}$$

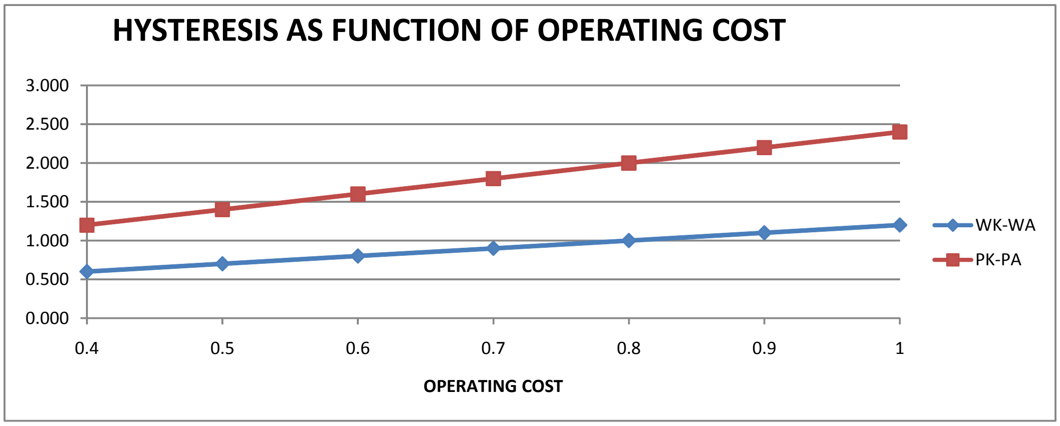
The Marshallian trigger prices for investment and abandonment are W_K and W_A. The former is the usual full cost but the latter differs from variable cost C, because we now have abandonment costs. At a price between these limits, an idle firm does not invest and an active firm does not exit. Equations (8.18) and (8.19) show that uncertainty widens this Marshallian range of inaction, if $(\delta/r)*(\beta_1/(\beta_1-1))>1$ and $(\delta/r)*(-\beta_2/(-\beta_2-1))<=1$.

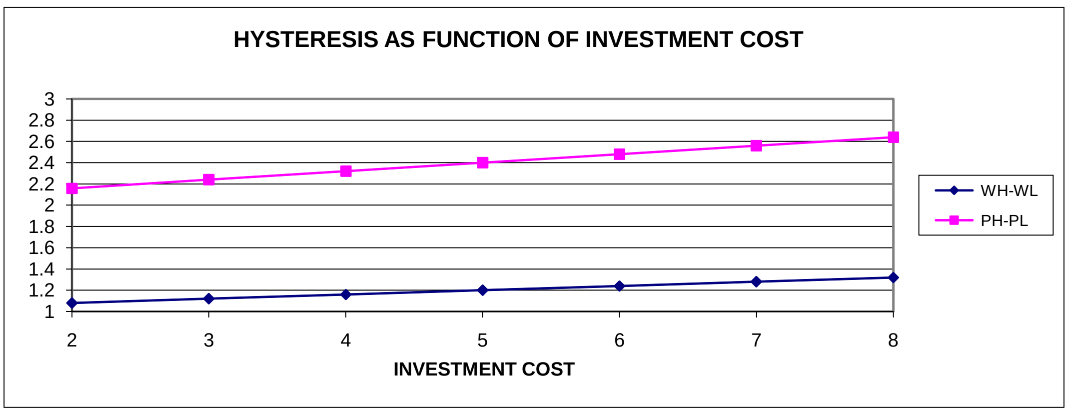
Figure 8.1

	A	B	C
1	**MARSHALLIAN ENTER/EXIT PRICES**		
2	**SIMPLE OPTIONAL ENTER/EXIT PRICES**		
3	**INPUT**		
4	**P**	**2**	
5	**Operating cost C**	**1**	
6	**Investment K**	**5**	
7	**Abandon A**	**25**	
8	**Interest Rate**	**0.04**	
9	**Asset Yield**	**0.04**	
10	**Asset Growth**	**0.00**	
11	**Asset Volatility**	**0.20**	
12	**OUTPUT**		
13	β_1	2.00	0.5-(B8-B9)/(B11^2)+SQRT(((B8-B9)/(B11^2)-0.5)^2 + 2*B8/(B11^2))
14	β_2	-1.00	0.5-(B8-B9)/(B11^2)-SQRT(((B8-B9)/(B11^2)-0.5)^2 + 2*B8/(B11^2))
15	W_K	**1.200**	B5+B8*B6
16	W_A	**0.000**	IF((B5-B8*B7)>0,B5-B8*B7,0)
17	P_K	**2.400**	(B8-B10)/B8*B13/(B13-1)*B15
18	P_A	**0.000**	(B8-B10)/B8*(-B14/(-B14+1))*B16
19	W_K-W_A	**1.200**	B15-B16
20	P_K-P_A	**2.400**	B17-B18
21	(P_K-P_A)/(W_K-W_A)	**2.000**	B20/B19

Figure 8.1 shows that with C=1, r=4%, δ=4%, σ=20%, K=5, A=25, the hysteresis range is 1.20 with Marshall investment/abandonment rules, and 2.40 considering the real options to enter and exit given stochastic prices. The hysteresis ranges under both investment rules are sensitive to changes in the parameter values (except prices).

Figure 8.2

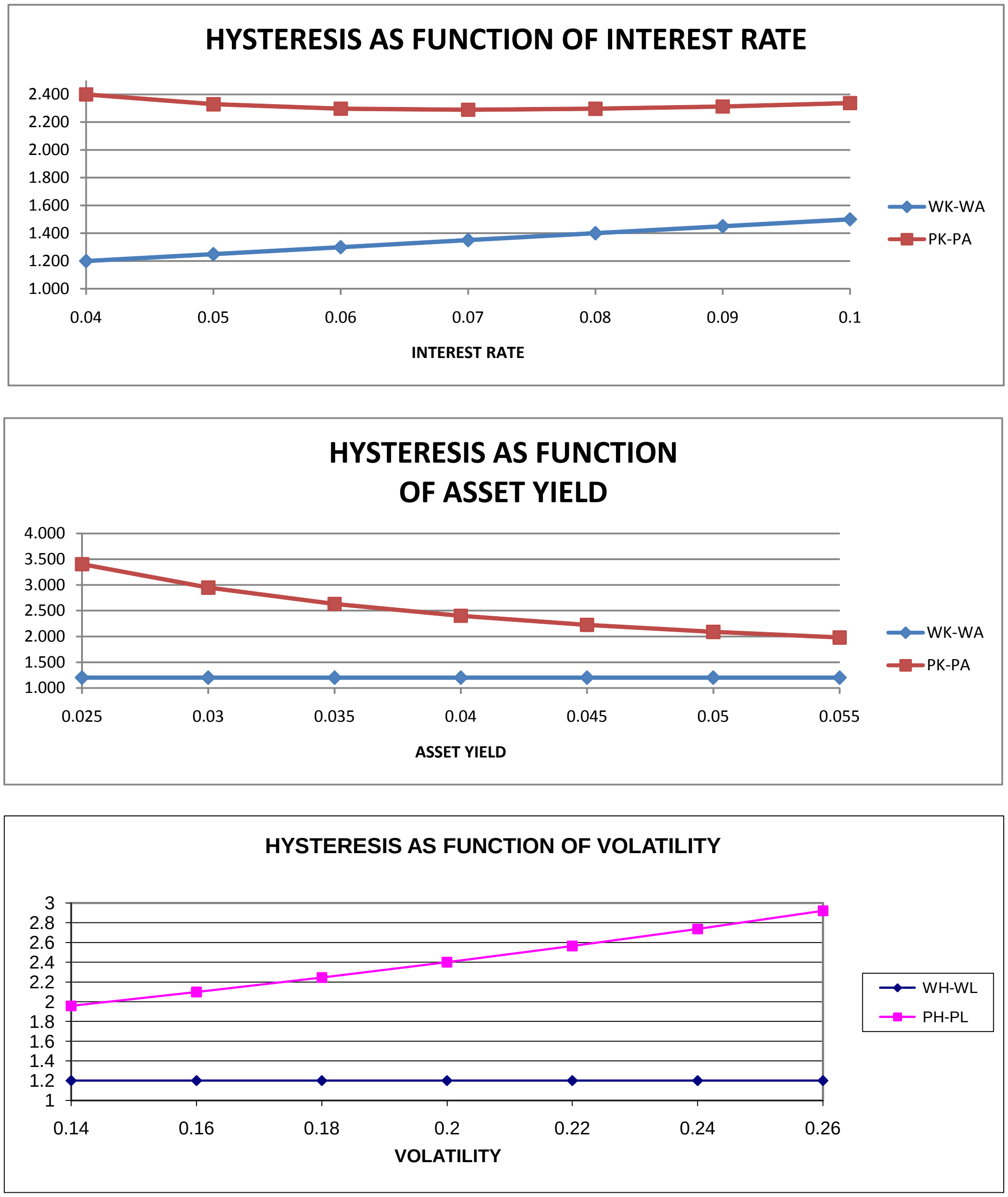




In Figure 8.2, the standard parameter values are C=1, K=5, A=25, r=4%, δ=4%, and σ=20%, assumed to be independent. Figure 8.2 shows that hysteresis increases as a function of C under the real option rules, and, of course, under Marshall, since W_K is C plus or minus a constant rK or rA. Hysteresis is a positive function of K under both rules. Note that C-rA<=0, if r>=.04.

Figure 8.3 shows that hysteresis is not always a positive function of interest rates under the stochastic rule, if δ=r-u and u is constant, but a negative function of payout, except under Marshall, since asset yield is not considered in that model

Figure 8.3



directly. Marshall hysteresis is insensitive to changes in asset volatility, since that model is deterministic, but real option hysteresis is highly sensitive. As Dixit (1989) notes in his footnote 9, stochastic hysteresis is greater than deterministic hysteresis if r/u >β.

The constraints for the previous analysis require essentially no abandonment, although it is assumed under both the stochastic and deterministic rules the abandonment trigger is 0. When there is an abandonment option and initially (C-rA)>0, these simple stochastic rules are not applicable. Figure 8.4 shows the numerical solution for the four equations with an abandonment option, using the Dixit (1989) parameters with abandonment cost A=0, with the derived stochastic triggers.

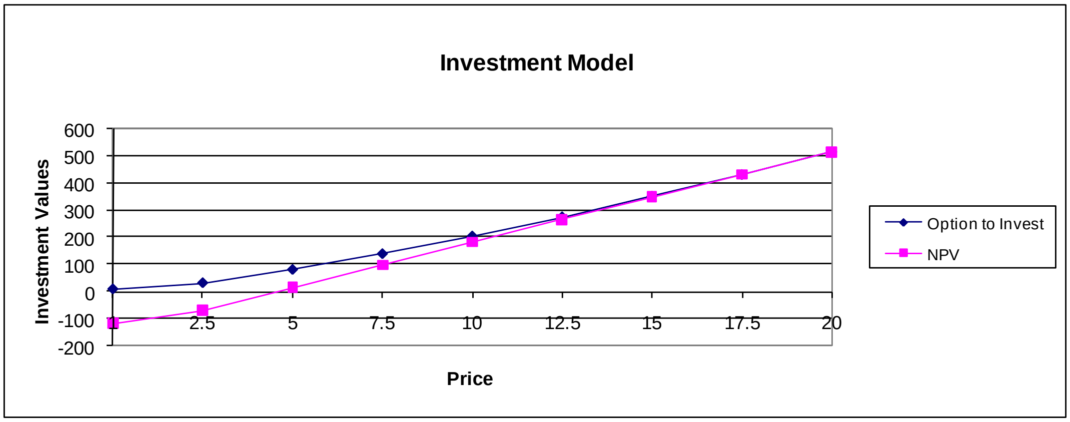
Figure 8.4

	A	B	C	D	E
1		**Entry - Exit (STOCHASTIC MODEL)**		**MARSHALLIAN TRIGGERS**	
2		INPUT		**INPUT**	
3	P=	2		**P**	**2**
4	K=	4		**Investment K**	**4**
5	C=	1		**Operating cost C**	**1**
6	A=	0		**Abandon A**	**0**
7	r=	0.025		**Interest Rate**	**0.025**
8	δ=	0.025		**Asset Yield**	**0.025**
9	σ=	0.1		**Asset Volatility**	**0.1**
10					
11	β1=	2.791	0.5-((B7-B8)/(B9^2))+SQRT(((((B7-B8)/(B9^2))-0.5)^2)+(2*(B7/(B9^2))))	β_1	2.79
12	β2=	-1.791	0.5-((B7-B8)/(B9^2))-SQRT(((((B7-B8)/(B9^2))-0.5)^2)+(2*(B7/(B9^2))))	β_2	-1.79
13		OUTPUT			
14	Vo(P)	44.053	B17*(B3^B11)	**OUTPUT**	
15	VK(P)	42.219	B18*(B3^B12)+(B3/B8)-(B5/B8)	**W_K**	**1.100**
16				**W_A**	**1.000**
17	A1	6.364		**W_K-W_A**	**0.100**
18	B2	7.681			
19	**P_K**	1.467		**P_K**	**1.714**
20	**P_A**	0.766		**P_A**	**0.642**
21	**P_K-P_A**	0.701		**P_K-P_A**	**1.072**
22	Eq.8.14	0.000	(-B17*(B19^B11))+(B18*(B19^B12))+(B19/B8)-(B5/B8)-B4		
23	Eq.8.15	0.000	(-B11*B17*(B19^(B11-1)))+(B12*B18*(B19^(B12-1)))+(1/B8)	**(P_K-P_A)/(W_K-W_A)**	**10.723**
24	Eq.8.16	-0.001	(-B17*(B20^B11))+(B18*(B20^B12))+(B20/B8)-(B5/B8)+B6		
25	Eq.8.17	0.000	(-B11*B17*(B20^(B11-1)))+(B12*B18*(B20^(B12-1)))+(1/B8)		
26	SOLVER	Set B22=0, and B23=B24=B25=B22, CHANGING B17:B20.		C-rA	1

Now that operating cost C > rA, the Marshallian exit trigger is the operating cost, but equations 8.18 and 8.19 are no longer valid, and there is no closed form solution for the stochastic case. The stochastic entry trigger is higher than the deterministic trigger (but lower than the hypothetical simple stochastic trigger). The stochastic exit trigger is lower than the deterministic trigger (but higher than the hypothetical simple stochastic trigger).

In the next several figures the standard parameters are: P=range of 1 to 20 (or to 9 in Figure 8.10), C=3, K=100, A=3, r=6%, δ=3% and σ=30%. Figure 8.5 shows that both the real option to invest and the Marshall valuation (NPV) are positive functions of price (at unitized volume), but the real option value is always positive. The option to invest is tangential to the intrinsic option value at the investment trigger price.

Figure 8.5



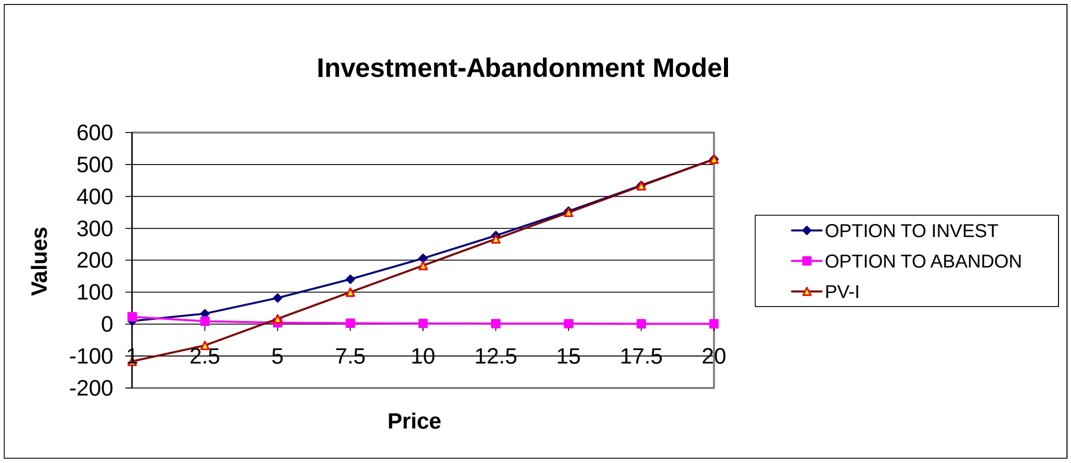
As shown in Figures 8.6 and 8.7, according to the basic investment-abandonment model, the values of the operating firm $V_K(P) = 6.664$ when P=1, and 617.833 when P=20, which is slightly more than 616.67, the present value (PV) of P discounted at the net cost of carry less operating costs discounted at the riskfree interest rate. The option to invest for the prospective asset owner is 9.567 even when P=1, but is equal to the intrinsic option value (PV-K) when $P>P_K$. The time value of the option to invest is equal to $V_0(P)$ when PV<K, declining to nil as P approaches P_K.

Figure 8.6

Investment-Abandonment Model

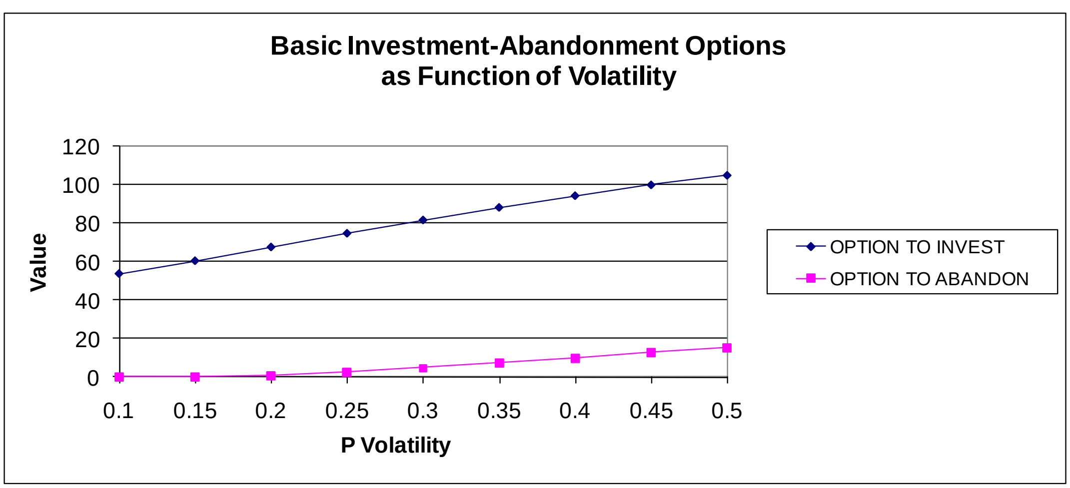
	INPUT									
P	Price	1	2.5	5	7.5	10	12.5	15	17.5	20
K	Initial Investment	100								
C	Operating Cost	3								
A	Abandonment Cost	3								
r	Riskless Rate	0.06								
δ	Cost of Carry	0.03								
σ	Volatility	0.3								
	OUTPUT									
β_1	Eq. 8.6	1.333								
β_2	Eq. 8.7	-1.000								
Vo(P)	Eq. 8.4	9.567	32.462	81.799	140.454	206.120	277.545	353.923	434.681	516.667
VK(P)	Eq. 8.5	6.664	42.665	121.333	203.111	285.666	368.533	451.555	534.666	617.833
A1	Value Coefficient of the Option to Invest	9.563								
B2	Value Coefficient of the Option to Abandon	23.330								
P(K)	Price Trigger for New Investment	17.724								
P(A)	Price Trigger to Abandon	1.073								
	PV-K	-116.667	-66.667	16.667	100.000	183.333	266.667	350.000	433.333	516.667
	TIME VALUE OF OPTION TO INVEST	9.567	32.462	65.132	40.454	22.787	10.879	3.923	1.348	0.000

Figure 8.7



However, consistent with general call and put option pricing, both the option to invest and the option to abandon increase with increases in the expected P volatility, as shown in Figure 8.8 (with P=5, slightly "in the money" for the option to invest). The optimal trigger for exercising the option to invest increases, and the trigger for exercising the option to abandon decreases, with increases in expected P volatility, given these parameters.

Figure 8.8



Note this model does not allow for a time lag between commissioning and the completed construction of a new asset, or for more active (enlarged flexibility) management of the asset. In this simplistic model, the options of the operating company are confined to moving to one alternative state, which clearly cannot be sufficient to accommodate the complexity of decision-making within a dynamic environment.

8.4 MULTIPLE SCALE OPTIONS

Let us consider more appropriate and sequentially ordered management actions after the initial investment. A firm converts from being idle (with, however, a proprietary option to build), to investing in and operating a facility. Then if P falls, there is an option to contract, reducing costs (and reducing services and prices). If P then increases, the next option is to revert to full service. If P falls instead, the next option is abandonment, or switching to an alternative use. Thus up to four options are considered, each with trigger prices, exercise costs, and usually different prices and operating costs in each state (see Figure 8.9, which includes additional actions of suspension, expansion, and reversion to previous states).

After providing the appropriate differential equation for each state, a numerical solution is proposed for the set of equations, which depends on the coefficients for state valuation and derived price triggers. The "four" states model developed in this section encompasses an orderly progression from the states of being idle, fully operational, contracted or abandoned[4]. Then there is one reversion from contracted to the previous state.

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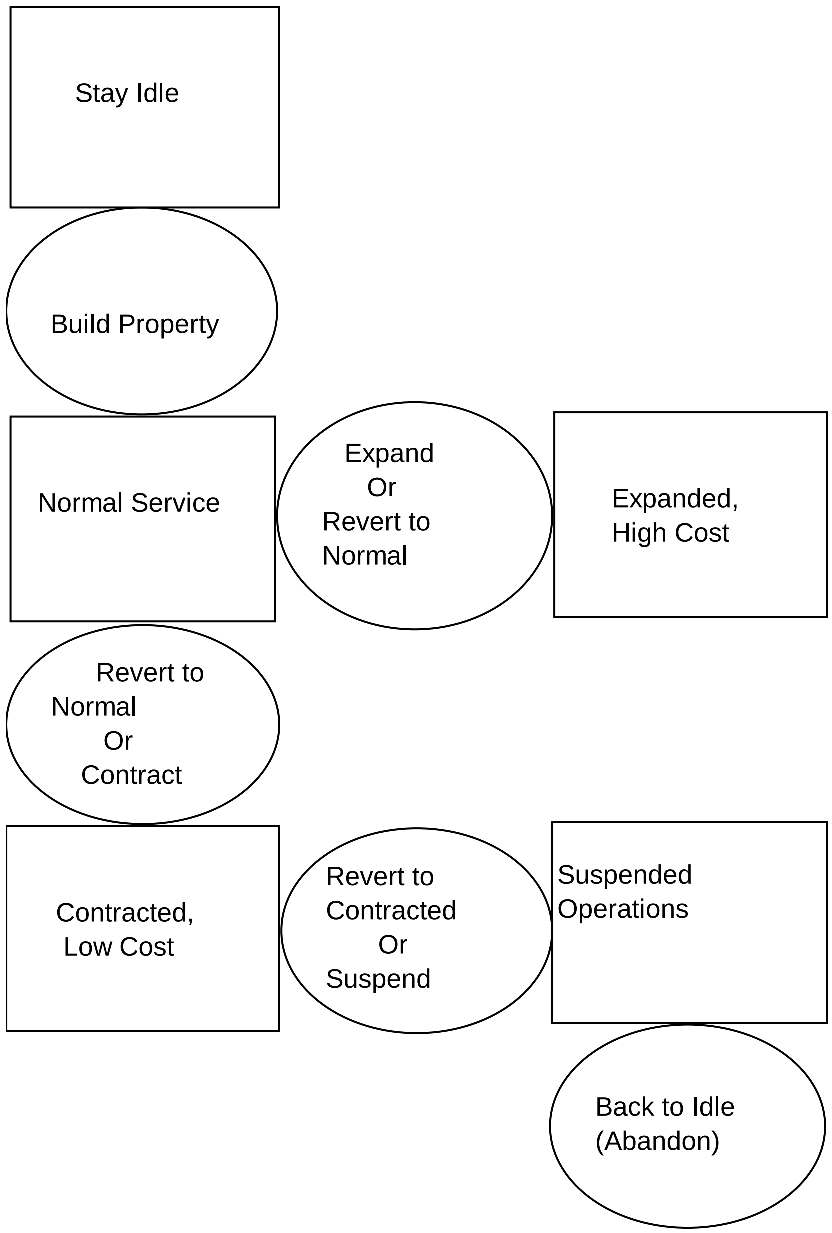
[4] "Complete" states might include selling properties (or ships or planes), or acquiring properties, as well as shifting from idle to suspension, contraction or expansion directly, and other reversions to and from other states, if feasible and economic.

Figure 8.9

Asset Strategic States and Actions



Additional symbols are:

E=the downsizing investment costs;

R = lump-sum cost of reverting to the previous state;

C_1=annual operating cost in the contracted state;

ς =price (P) multiplier in the contracted state, ς <1;

P_C = P threshold that motivates the owner to contract, reducing costs; and

P_R = P threshold that justifies reversion to full service operations.

There is an additional valuation function, $V_C(P)$ = value of a contracted operating facility, and an additional differential equation that the valuation functions must satisfy:

CONTRACT

$$\tfrac{1}{2}\sigma^2 P^2 V_C''(P) + (r-\delta) P V_C'(P) - r V_C(P) + \varsigma P - C_1 = 0 \tag{8.22}$$

The solution for this equation is:

$$V_C(P) = D_1 P^{\beta_1} + D_2 P^{\beta_2} + \varsigma P / \delta - C_1 / r \tag{8.23}$$

In equations (8.4), (8.12) and (8.23), A_1 is the value coefficient of the option to invest in the "full service" state (which itself includes two options), B_2 is the value coefficient of the option to contract, D_1 is the value coefficient of the option to revert to the full service operating state and D_2 is the value coefficient of the option to abandon. All of the coefficients are constrained to be positive (purchased or held, versus written, options always have non-negative values). In equation (8.23), the last term represents the perpetual net value of contracted state cash flows. Each of these four options has an exercise cost: the option to invest K, the option to contract E, the option to revert to full service from the contracted state R, and the option to abandon A. It is expected that there is a descending order of the triggers that signal the optimal actions for moving to each state, so that $P_K>P_R>P_C>P_A>0$, although the level and order of the normal service and reduced service reversion triggers depend on the other parameters including the exercise costs of the reversion. Optimal switching to four alternative states is based on four optimal triggers. Investment in new property takes place as soon as P rises to P_K. If the firm already holds a property in operation and P falls to a lower level P_C, management will choose to contract, that is reduce costs to C_1 , and P is reduced to ςP because of the reduced service or quality. If P falls even further to a lower threshold P_A, the property owner will be better off by abandoning the property. Alternatively, if P rises to a level P_R rather than fall to P_A, the operation will revert to full service. Assume that the property is not yet constructed and P starts from a low initial value. The investment costs K and downsizing costs are constant, as are the operating costs C_1 and C, with $C_1 < C$. Furthermore it is assumed that the full service price will be reduced to ςP in the contracted state, and that first the normal operating state must be constructed. The firm is idle initially over the range (0, P_K) and in the contracted state over the range (P_A, P_R). The value matching condition is

$$B_2 P_C^{\beta_2} + P_C / \delta - C / r = D_1 P_C^{\beta_1} + D_2 P_C^{\beta_2} + \varsigma P_C / \delta - C_1 / r - E \qquad (8.24)$$

and the smooth pasting condition is

$$\beta_2 B_2 P_C^{\beta_2 - 1} + 1/\delta = \beta_1 D_1 P_C^{\beta_1 - 1} + \beta_2 D_2 P_C^{\beta_2 - 1} + \varsigma / \delta \qquad (8.25)$$

which when simplified are equations (8.26) and (8.27).

$$- D_1 P_C^{\beta_1} + (B_2 - D_2) P_C^{\beta_2} + (1 - \varsigma) P_C / \delta - (C - C_1) / r + E = 0 \qquad (8.26)$$

$$- \beta_1 D_1 P_C^{\beta_1 - 1} + \beta_2 (B_2 - D_2) P_C^{\beta_2 - 1} + (1 - \varsigma) / \delta = 0 \qquad (8.27)$$

$$D_1 P_R^{\beta_1} + (D_2 - B_2) P_R^{\beta_2} - (1 - \varsigma) P_R / \delta + (C - C_1) / r + R = 0 \qquad (8.28)$$

$$\beta_1 D_1 P_R^{\beta_1 - 1} + \beta_2 (D_2 - B_2) P_R^{\beta_2 - 1} - (1 - \varsigma) / \delta = 0 \qquad (8.29)$$

$$(D_1 - A_1) P_A^{\beta_1} + D_2 P_A^{\beta_2} + \varsigma P_A / \delta - C_1 / r + A = 0 \qquad (8.30)$$

$$\beta_1 (D_1 - A_1) P_A^{\beta_1 - 1} + \beta_2 D_2 P_A^{\beta_2 - 1} + \varsigma / \delta = 0 \qquad (8.31)$$

In addition to equations (8.16) and (8.17), equations (8.28) and (8.29) derive from the conditions at the reversion threshold, whereas equations (8.30) and (8.31) are based on the abandonment threshold. The numerical solution should satisfy $P_K > P_R > P_C > P_A > 0$, and the option value coefficients should be positive.

Not surprisingly, in the four options model the trigger price thresholds that justify new investment are similar to the ones obtained from the two options model. Figure

8.10 shows that the company should invest in new property, as soon as P rises above 17.720 which is slightly lower than in the simple two option model.

Figure 8.10

	A	B	C	D	E	F	G	H	I	J	K
1			**Invest, Revert, Contract and Abandonment**								
2		INPUT									
3	P	**Price**	**1.00**	**2.00**	**3.00**	**4.00**	**5.00**	**6.00**	**7.00**	**8.00**	**9.00**
4	K	Initial Investment	100.00								
5	C	Operating Cost	3.00								
6	r	Riskless Rate	0.06								
7	δ	Cost of Carry	0.03								
8	σ	Volatility of P	0.30								
9	E	Cost of Contracting	5.00								
10	A	Abandonment Cost	3.00								
11	R	Cost for Reversion:FS from Contraction	2.00								
12	C1	Reduced Operating Cost in Contraction	1.50								
13	ς	Contraction Multiplier	0.60								
14	β1	Eq. 8.6	1.333								
15	β2	Eq. 8.7	-1.000								
16											
17	V0(P)	Eq. 8.4	9.564	24.100	41.381	60.728	81.771	104.274	128.067	153.024	179.046
18	VK(P)	Eq. 8.12	4.013	27.006	56.893	88.503	120.803	153.447	186.288	219.252	252.298
19	VC(P)	Eq.8.23	7.643	31.611	59.558	89.233	120.065	151.799	184.292	217.451	251.206
20											
21	A1	Value Coefficient of the Option to Invest	9.564								
22	B2	Value Coefficient of the Option to Contract	20.679								
23	D1	Value Coefficient of the Option to Revert:FS1	5.094								
24	D2	Value Coefficient of the Option to Abandon	7.549								
25	P(K)	Price Trigger for New Entry	17.720								
26	P(R)	Price Trigger to Revert	7.134								
27	'P'('C)	Price Trigger to Contract	1.549								
28	P(A)	Price Trigger to Abandon	0.717								
29											
30		Eq.8.16	0.0	(C24)*(C28^C17)-C25*(C28^C18)-(C28/C10)+(C8/C9)+C7							
31		Eq.8.17	0.0	C17*(C24)*(C28^(C17-1))-C18*C25*(C28^(C18-1))-(1/C10)							
32		Eq.8.26	0.0	(-C26)*(C30^C17)+(C25-C27)*(C30^C18)+((1-C16)*C30)/C10-((C8-C15)/C9)+C12							
33		Eq.8.27	0.0	C17*(-C26)*(C30^(C17-1))+C18*(C25-C27)*(C30^(C18-1))+(1-C16)/C10							
34		Eq.8.28	0.0	(C26)*(C29^C17)+(C27-C25)*(C29^C18)-(1-C16)*C29/C10+(C8-C15)/C9+C14							
35		Eq.8.29	0.0	C17*(C26)*(C29^(C17-1))+C18*(C27-C25)*(C29^(C18-1))-(1-C16)/C10							
36		Eq.8.30	0.0	(C26-C24)*(C31^C17)+C27*(C31^C18)+((C16*C31)/C10)-C15/C9+C13							
37		Eq.8.31	0.0	C17*(C26-C24)*(C31^(C17-1))+C18*C27*(C31^(C18-1))+(C16/C10)							
38		SOLVER: SET C41=0, CHANGING C24:C31	0.0								
39	V0(P)	Eq. 8.4	C24*(C6^C17)								
40	VK(P)	Eq. 8.12	C25*(C6^C18)+(C6/C10)-(C8/C9)								
41	VC(P)	Eq.8.23	C26*(C6^C17)+C27*(C6^C18)+(C16*C6/C10)-(C15/C9)								

Solve the eight simultaneous equations in cells C33:C40 by setting the absolute sum in cell C41=0, by changing the unknowns in cells C24:C31. P need to fall below the exit thresholds that have been identified previously to motivate abandonment. This reflects the fact that the property owner now has the additional option to contract and operate a reduced service for the property prior to abandonment. A restaurant might add tables in less favourable locations; an airline might bear with more customer unhappiness as seats are smaller, free meals eliminated, and flights even more overbooked; a hotel might reduce service staff (downgrade to Three Star from Four Star) while reducing room rates. There are new derived threshold price triggers for the contracting and reversion to normal operating state decisions. Note that the coefficient of the abandonment option is sharply reduced, but the coefficient for the contracting option is large. The value function of the contracted state includes the option to revert to normal operation or abandonment at the optimal time. At high current P, there is little option value for downsizing from the normal operation to a contracted state, or for the contracted operation to abandonment, as expected. However, at low current P, at these reversion, contracting and abandonment costs, there may be significant option value for these states.

Increases in operating cost result in slightly decreased real call (investment, reversion) option values, increased real contracting value, and increases in reversion and contracting triggers. Increases in the reduced operating cost in the contracting state decrease the contracting option, and increase the abandonment option and trigger. Increases in the contraction multiple result in increased contracting option value and trigger, and decreased reversion and abandonment option values. Increases in K reduce the option to invest, and increase the investment trigger.

Increases in interest rates, the net cost of carry and volatility result in changes in β_1 and β_2. Interest rate increases reduce β_1 and increase β_2, while net carrying costs have the opposite effect. Increases in volatility reduce β_1 and increase β_2. Increases in interest rates increase call (invest, revert) options, and significantly decrease put (contract, abandon) options, with an opposite effect for net carrying costs. Increases in volatility increase all option values, especially contracting and abandonment, but increase call triggers and decrease put triggers.

SUMMARY

This chapter develops a real perpetual American option model, extended to multiple states, starting with entry plus exit, then adding contracting and reversion to a full operating state. The solution for a single state investment choice can be easily calculated, as can the Marshallian investment rules. More complex states require numerical solutions, which are feasible in spreadsheets.

EXERCISES

EXERCISE 8.1 Curiously the Real Options (RO) class exit cost will always exceed the value of the class on-going cost. If you take the class, there is no exit. Once committed, forever committed. Current perpetual benefits to a student taking RO are believed to be (P) 2 per annum, class annual cost (C) is 1 in effort in perpetuity given the mental and physical stress, benefit volatility=20%, benefit yield 4% (δ), riskless rate 4% (r), once off sunk entry cost is 5 (K) and exit 25 (A). Should you enter the class now? What if you were following Professor Marshall?

EXERCISE 8.2 Prove that your solution in Exercise 8.1 solves the idle equation 8.4 (including 8.10 and 8.11).

EXERCISE 8.3 Show the real option versus Marshallian hysteresis over a range of operating costs from .8 to 1.0? Why might you be reluctant to change (if you could) over a wider range with higher costs if you were aware of real options theory than if you were stuck in Cambridge with Professor Marshall?

PROBLEMS

PROBLEM 8.4 Suppose you have the option of entering or leaving the RO class at a pre-specified entering/exiting cost. Arriving at the class, view the quality and quantity of the lectures, your prospects and entertainment value and alternative uses of your time and effort. After all, if you leave the class, and the supposed quality of the lectures improves, and quantity declines, or your prospects and alternatives change, you can try this class again. Current perpetual benefits to a student taking RO are believed to be 2, class cost .4 in effort in perpetuity given the mental and physical stress, entry cost 5, exit 2, benefit volatility=20%, benefit yield 4%, riskless rate 4%. When should you enter the class? If in the class, if the perceived benefits decline, when should you exit?

PROBLEM 8.5 You have the opportunity of investing in a 100 room "suite-hotel" for $100,000 per room. Yearly rents are currently $10,000 per room, 90% occupancy is anticipated, and annual operating costs are $3000 per room. Expected volatility of annual rent per room capacity is 30%, interest rates 10%, and the required property yield is 5%. This facility can be switched once into a permanent residence accommodation worth $75,000 per room, but the switching-refurbishment costs are $20,000 per room. What is this opportunity worth and at what expected hotel rental level should you take up the opportunity ? At what rental level should you switch the facility use?

PROBLEM 8.6 What if there is the feasible alternative of downsizing the hotel facility to half capacity (costs $15,000 per room), with operating costs reduced to $1000 per room, with the possible reversion to full capacity (which costs $10,000 per room), but it costs $5000 per room to abandon the hotel?

APPENDIX 8A

If there is no abandonment option, then B_2=0, equations 8.14 and 8.15 are:

$$A_1 P_K^{\beta_1} - P_K/\delta + C/r + K = 0 \qquad \text{(A1)}$$

$$\beta_1 A_1 P_K^{\beta_1 - 1} - 1/\delta = 0 \qquad \text{(A2)}$$

From A1

$$A_1 = (P_K/\delta - C/r - K)/P_K^{\beta_1} \qquad \text{(A3)}$$

Substitute A3 into A2

$$\beta_1[(P_K/\delta - C/r - K)/P_K^{\beta_1}]P_K^{\beta_1 - 1} = \frac{1}{\delta} \qquad \text{(A4)}$$

$$-\beta_1\delta(C/r + K) = P_K(1 - \beta_1) \qquad \text{(A5)}$$

Note that $1/P_K = \dfrac{P_K^{\beta_1 - 1}}{P_K^{\beta_1}}$, $\dfrac{-\beta_1}{1-\beta_1} = \dfrac{\beta_1}{\beta_1 - 1}$, *and* $W_K/r = C/r + K$

$$P_K = \frac{\beta_1}{\beta_1 - 1}\frac{\delta}{r}(C + rK) \qquad \text{(8.18)}$$

Some of the partial derivatives are easy.

$$\frac{\partial P_K}{\partial C} = \frac{\beta_1}{\beta_1 - 1}\frac{\delta}{r} \qquad \text{(A6)}$$

$$\frac{\partial P_K}{\partial K} = \frac{\beta_1\delta}{\beta_1 - 1} \qquad \text{(A7)}$$